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SHORTER COMMUNICATIONS

THERMAL CONSTRICTION RESISTANCE DUE TO NON-UNIFORM SURFACE CONDITIONS; CONTACT RESISTANCE AT NON-UNIFORM INTERFACE PRESSURE

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NOMENCLATURE

A, area;
a, radius of the contour area; half the width of the contour strip;
b, radius of the elemental heat channel; half the width of the heat channel;
f, h/h_{av};
H, hardness of the softer material;
h, heat transfer coefficient; for dissimilar metals in contact

$$K = \frac{2K_1K_2}{K_1 + K_2};$$

J_n, Bessel function of order n;
k, thermal conductivity;
P, interface pressure;
Q, total heat rate;
R, thermal resistance;
R_c, constriction thermal resistance;
r, coordinate;
T, temperature;
$\tan\theta$, average of absolute slope of surface irregularities $\tan\theta = (\tan^2\theta_1 + \tan^2\theta_2)^{\frac{1}{2}}$;
x, coordinate;
z, coordinate.

Greek letters
λ, r/b; x/b;
v_n, eigenvalues; equation (6);
σ, r.m.s. of contacting surface, $\sigma = (\sigma_1^2 + \sigma_2^2)^{\frac{1}{2}}$.

1. INTRODUCTION

HEAT flow through a material with a non-uniform heat flux over its surfaces is always associated with a thermal resistance known as a constriction resistance. The constriction resistance is caused physically by the heat flow redistribution in the material so that the flow could conform with the non-uniform heat flux at the surface. This resistance is significant only if the characteristic length representing non-uniform conditions over the surface is less than or equal to the depth of the material where the constriction takes place. The most common example of the constriction resistance is the so called thermal contact resistance. Less common, but also very significant, is construction resistance in dropwise condensation where this type of the resistance, for example, could account for about 80 per cent of the total resistance (in case of stainless steel as condensing surface [1]). The same phenomena could be significant for convective heat transfer to liquid metals, when gas bubbles are entrained on the heat-transfer surface [2].

The results of this work are confined to contact resistance and specifically to the macroscopic contact resistance for the case of non-uniform interface pressure distribution. The approach employed here, however, is more general and can be used to solve most of the problems involving constriction resistance.

The major theoretical contributions in the area of thermal contact resistance dealt with a simple contact [3–8] multiple contacts [3, 5, 7–10], directional effects [11, 12] and others. A good bibliography on the subject, including important experimental work and significant publications beyond those listed above, is reported in [13].

A non-uniform pressure distribution at metallic interfaces, although a common occurrence in practice, was never considered explicitly in calculation of the contact resistance. Non-uniform pressure will be present in the case of a deviation of flatness (waviness) in one or both of the contacting surfaces; it could also arise due to the nature of loading. In either case, the contact points distribution and the distribution of actual contact area will be non-uniform (in excess of the randomness effect).

For two surfaces in nominal contact, the actual contact occurs only at a number of discreet contact points. If the apparent interface pressure is uniform, those points are distributed randomly over the surface. The resistance caused by random distribution of contact points is usually called microscopic contact resistance. In the presence of waviness, those points would appear in clusters, or more generally, would be non-uniformly distributed. The resistance caused by clustering is called macroscopic contact resistance. Holm [3] and later Kragelski [14], and Clausing [6], proposed a formula for the calculation of the contact resistance for this case. The relation is based on superposition of the two resistances. The macroscopic resistance is calculated from an expression for a single big contact, taking the contour area which contains all the microscopic contacts inside it, to represent the macroscopic contact. A similar expression was also derived by Greenwood [10]. The difference between his and Holm's expression is only in the assumed macroscopic conditions over the contour area: the latter assumes constant temperature and the former, constant heat flux over the contour area. Both formulae, in addition to some other limitations imposed by the physical model employed in their derivation, required the knowledge of the contour area. Moreover, if the contour area is the same as the apparent area, i.e. the contact points are distributed over the whole interface, the macroscopic resistance, according to the suggested expressions, would be zero regardless of the distribution details inside the contour area. Physically, of course, it should be present in this case, too.

In this work, the concept of the contour area is eliminated and the macroscopic construction is related directly to non-uniformity of the macroscopic heat flux and specifically, to the pressure distribution over the interface. The result is incorporated into an expression which relates the overall contact resistance (microscopic and macroscopic) to the pressure distribution and surface properties.

2. CONSTRICTION RESISTANCE DUE TO NON-UNIFORM HEAT FLUX DISTRIBUTION

Consider the flow of heat through a solid (Fig. 1). At the surface $z = 0$, there is non-uniform heat-transfer coefficient h. Far from the surface (large z) the temperature distribution is one-dimensional. Let T_c represent the local surface temperature and T_0 constant temperature of the environment. The heat rate at the surface is then

$$Q = \int_A h(T_0 - T_c)\,\mathrm{d}A. \quad (1)$$

The surface temperature T_c would be non-uniform (higher at places of higher heat flux, since the gradient there would be higher, Fig. 1b). The average flux over the heat-transfer surface follows from (1) as

$$\frac{Q}{A} = T_0 h_{av} - \frac{1}{A}\int_A T_c h\,\mathrm{d}A = (T_0 - T_s)\,h_{av} - \frac{1}{A}\int_A (T_c - T_s)\,h\,\mathrm{d}A \quad (2)$$

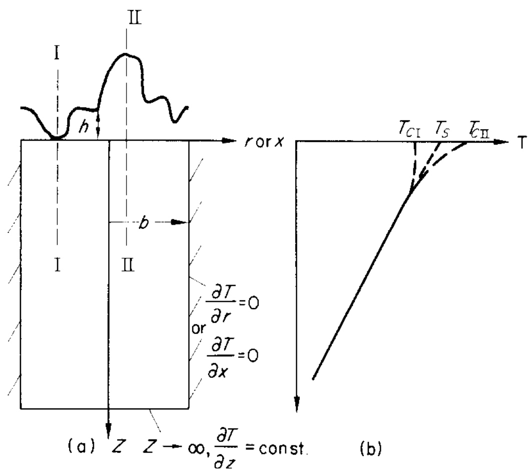

FIG. 1.

where $h_{av} \equiv (1/A)\int h\,\mathrm{d}A$, and T_s is a constant hypothetical surface temperature obtained at $z = 0$ by the extrapolation of the linear temperature profile existing far from the surface (Fig. 1b.)

Defining the total resistance from the surface to the environment as

$$R \equiv \frac{T_0 - T_s}{Q/A}, \quad (3)$$

one can write from relation (2) the following

$$R = \frac{1}{h_{av}} + R_c, \quad \text{where} \quad R_c \equiv \frac{1}{Q}\int_A \frac{h}{h_{av}}(T_c - T_s)\,\mathrm{d}A. \quad (4)$$

Equation (4) defines the constriction resistance R_c. It can be seen from relation (4) that $1/h_{av}$ is not the only resistance at the surface. The value of R_c is always positive since $T_c - T_s$ (see Fig. 1b) is higher for higher values of h/h_{av}. R_c goes to zero when $T_c \to T_s$ everywhere, and that would be the case either for uniform h or an infinite conductivity of the surface material in the lateral direction.

(a) *Constriction resistance in cylindrical coordinates*

Consider a heat flow through a cylinder of radius b, Fig. 1; at $z = 0$ heat transfer coefficient is given as a function of radius $h = h(r)$. In order to get a first approximation for the constriction resistance due to the non-uniform h, it will be assumed that the value of the local heat flux at $z = 0$ is proportional to the local value of heat-transfer coefficient, i.e.

$$-\frac{k(\partial T/\partial z)_{z=0}}{Q/A} \simeq \frac{h}{h_{av}}. \quad (5)$$

The temperature distribution inside the cylinder, and subsequently the temperature distribution at surface $z = 0\,(T_c)$ could be obtained by solving the Laplace differential equation with the appropriate boundary conditions, including condition (5), arriving at the following expression

$$T_c - T_s = \frac{2Q}{\pi bk} \sum_{n=1}^{\infty} \frac{\int_0^1 \lambda f(\lambda)\, J_0(\nu_n\lambda)\, d\lambda}{\nu_n J_0^2(\nu_n)} J_0(\nu_n\lambda) \tag{6}$$

where $\lambda \equiv r/b$, $f(\lambda) \equiv h/h_{av}$ and eigenvalues ν_n are the roots of the following equation $J_1(\nu_n) = 0$.

The value for the constriction resistance now can be evaluated from (4) and (6), yielding the following expression for R_c

$$R_c = 4\frac{b}{k} \sum_{n=1}^{\infty} \frac{[\int_0^1 \lambda f(\lambda)\, J_0(\nu_n\lambda)\, d\lambda]^2}{\nu_n J_0^2(\nu_n)}. \tag{7}$$

In the case of step distribution of h: $h = h_0$, $0 < r < a$ and $h = 0$, $a < r < b$, relation (7) changes into

$$\frac{kR_c}{b} = 4\left(\frac{b}{a}\right) \sum_{n=1}^{\infty} \frac{J_1^2[\nu_n(a/b)]}{\nu_n^3 J_0^2(\nu_n)} \approx \frac{8}{3\pi}\left(\frac{b}{a}\right)\left(1 - \frac{a}{b}\right)^{1\cdot 5}.$$

For parabolic changes of h, i.e. $h = h_0(1 - r^2/a^2)$, $0 < r < a$, $h = 0$, $a < r < b$, (7) yields the following

$$\frac{kR_c}{b} = 256\left(\frac{b}{a}\right)^4 \sum_{n=1}^{\infty} \frac{J_2^2[\nu_n(a/b)]}{\nu_n^5 J_0^2(\nu_n)} \approx 1\cdot 9\left(\frac{b}{a}\right)^4 J_2(3\cdot 83\,a/b)$$

(b) *Constriction resistance in Cartesian geometry*

In case when the waviness is two dimensional, rather than circular, or when the nature of the loading is such as to produce the interface pressure variation only in one direction, one has to express the constriction resistance in the Cartesian coordinates. Let heat transfer coefficient h, at the surface $z = 0$, see Fig. 1, be non-uniform in x-direction only, then the identical procedure with the one used in the previous section yields the following expression for the constriction resistance

$$R_c = \frac{2b}{k} \sum_{n=1}^{\infty} \frac{1}{n}\left[\int_0^1 f(\lambda) \cos(n\pi\lambda)\, d\lambda\right]^2 \tag{8}$$

where here $f(\lambda) \equiv h(x)/h_{av}$ and $\lambda \equiv x/b$.

From (8) one can calculate explicitly the constriction resistance for a given specific distribution of $h(x)$.

The results for R_c given in this work represents an upper bound for the respective non-uniformities in h, due to the approximation in the boundary condition at $z = 0$, equation (5). However, if a particular non-uniformity were representing the heat flux non-uniformity, the developed relations would represent the exact solutions. The concept of h was used here only in order to approach the introduction of the constriction resistance through a more familiar way and show that $1/h_{av}$ is not the only resistance in the considered system. The concept would fail when the non-confirmity is caused by non-uniform fluid temperature rather than non-uniform h. Therefore, more general approach would be the consideration of non-uniform heat flux at the surface as the direct cause of the constriction resistance, regardless of the source of the non-uniformity, (e.g. non-uniformity h, non-uniform fluid temperature).

3. CONTACT RESISTANCE DUE TO NON-UNIFORM PRESSURE

Contact resistance due to the microscopic constriction (roughness effect) was considered in details in [7].

With assumed Gaussian distribution of surface heights, the microscopic contact conductance was related to the interface pressure, surfaces, characteristics and the hardness of the softer material in contact as

$$h_c = 1\cdot 45 \frac{\tan\theta}{\sigma} k \left(\frac{P}{H}\right)^{0\cdot 985}. \tag{9}$$

The parameters in the above equation are introduced in the nomenclature.

Relation (9), as written above, is applicable for contact in a vacuum. Employing an approximate, but generally accepted approach, one can modify expression (9) by simply adding to it conductance through the interfacial fluid in order to account for presence of the fluid.

The macroscopic part of the contact resistance as stated earlier, is caused by the distribution of local microscopic resistance. Consequently, with a known distribution of the interface pressure one can, from the presented relation, express the total resistance as a function of the interface pressure distribution and other pertinent parameters.

Proceeding in this direction one obtains from (9) and (7) the following relation for the total resistance [microscopic and macroscopic for cases where $P = P(r)$]:

$$\left.\begin{aligned} R = {} & 0\cdot 345 \frac{\sigma}{k \tan\theta}\left[\int_0^1 \lambda \left(\frac{P}{H}\right)^{0\cdot 985} d\lambda\right]^{-1} \\ & + 8\frac{b}{k} \sum^{\infty} \frac{[\int_0^1 \lambda (P/P_{av})^{0\cdot 985} J_0(\nu_n\lambda)\, d\lambda]^2}{\nu_n J_0^2(\nu_n)} \end{aligned}\right\} \tag{10}$$

$\lambda \equiv r/b$, k is defined by equation (17), $P_{av} \equiv 2\int_0^1 \lambda\, P\, d\lambda$, and ν_n are the roots of $J_1(\nu_n) = 0$.

For the cases where $P = P(x)$, from (9) and (8) follows:

$$\left. \begin{aligned} R = 0{\cdot}689 \frac{\sigma}{k \tan \theta} \left[\int_0^1 \left(\frac{P}{H} \right)^{0{\cdot}985} \mathrm{d}\lambda \right]^{-1} \\ + 4 \frac{b}{k} \sum_{n=1}^{\infty} \frac{1}{n} \left[\int_0^1 \left(\frac{P}{P_{av}} \right)^{0{\cdot}985} \cos(n\pi\lambda)\, \mathrm{d}\lambda \right]^2 \\ \text{Here, } \lambda \equiv x/b \text{ and } P_{av} = \int_0^1 P\, \mathrm{d}\lambda. \end{aligned} \right\} \quad (11)$$

Equations (10) and (11), as written above, are valid only for contacts in a vacuum. For contacts in a fluid environment modification of relation (16) should be made before its substitution in equations (9) or (12).

The derived relations are general in a sense that they do not require the knowledge of a contour area and they can be applied for any pressure distribution at the interface for which $P = P(r)$ or $P = P(x)$, respectively. For a special case of step distribution one would get from the above a simple known superposition formula.

We like to point out here that in cases where microscopic resistance is negligible compared with the macroscopic constriction, i.e. $h \to \infty$ over the contour area (which is in this case clearly defined) and $h = 0$ elsewhere, one gets for the macroscopic constriction resistance, in the case of radial symmetry (see [6, 8]) the following

$$R_c \approx \frac{\pi b^2}{2ka} \left(1 - \frac{a}{b} \right)^{1{\cdot}5},$$

where a is the radius of the contour area.

4. CONCLUSIONS

In this work the value of thermal constriction resistance was explicitly related to the non-uniform conditions at the heat transfer surface.

The developed concept is applied directly for calculation of the macroscopic contact resistance caused by non-uniform contact points distribution at the interface. The obtained relations are valid for an arbitrary interface pressure distribution. The results are applicable directly only to contact in a vacuum, but could be modified in order to take into account affects of the presence of an interfacial fluid.

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